1.1 Exercise Set

1. Evaluate.

a) $|6|$ 6

b) $|12|$ 12

c) $|-6|$ 6

d) $|-12|$ 12

e) $-|2|$ -2

f) $|-2|$ 2

g) $-|-3|$ -3

h) $|-3|+|2|$ 5

i) $|-4|+|4|$ 8

j) $|-3|-|3|$ 0

k) $|-4|-|-2|$ 2

l) $|-4-(-2)|$ $|-4+2|$ $|-2|$ 2

m) $-|2|+|-2|$ $-2+2$ 0

n) $|-2|+|-2|$ $2+2$ 4

2. Insert the absolute value symbol (if needed) to make the equation true.

a) $|-3|-|(-4)|=-1$

b) $|-3|-(-4)=7$

c) $-3-|(-4)|=-7$

d) $-2-|(-3)|-(-4)=-1$

e) $|-2|-|(-3)|-(-4)=3$

f) $-2-|(-3)|-|-4)|=-9$

g) $|-2|-|(-3)|-|(-4)|=-5$

h) $-1+(-2)-(-3)=0$

i) $-1+|(-2)|-|-3)|=-2$

j) $-1+|(-2)|-(-3)=4$

3. Arrange from least to greatest.

a) $-|3|, |-3|, -|-2|, |-(-2)|$ $-3\ \ 3\ \ -2\ \ 2$ $-|3|, -|-2|, |-(-2)|, |-3|$

b) $-|2.\overline{5}|, |-2.\overline{5}|, -|2.5|, |-2.5|$ $-2.\overline{5}\ \ 2.\overline{5}\ \ -2.5\ \ 2.5$ $-|2.\overline{5}|, -|2.5|, |-2.5|, |-2.\overline{5}|$

c) $-|3-7|, |-(3-7)|, -|5-|2-2||, -|5|-|-2|$ $-4\ \ 4\ \ -3\ \ -7$ $-|5|-|-2|, -|3-7|, -|5-|-2||, |-(3-7)|$

1.2 Exercise Set

1. True or false.

$x = \pm 3$

a) The equations $x^2 = 9$ and $x = 3$ are equivalent. T / (F)

b) The equation $x^2 = -9$ has no real solutions. (T) / F

c) The equation $x^2 = 0$ has no solution. T / (F)

$x = 0$

d) The equation $x^3 = 27$ has solution $x = \pm 3$. $\sqrt[3]{27} = 3$ only T / (F)

e) The equation $\sqrt{x} = -4$ has no real solution. (T) / F

2. Solve for x.

a) $\sqrt{x^2} = \sqrt{4}$ $x = \pm 2$

b) $\sqrt{x^2} = \sqrt{-4}$ $\varnothing$

c) $\sqrt[3]{x^3} = \sqrt[3]{8}$ $x = 2$

d) $\sqrt[3]{x^3} = \sqrt[3]{-8}$ $x = -2$

e) $\sqrt[4]{x^4} = \sqrt[4]{16}$ $x = \pm 2$

f) $\sqrt[4]{x^4} = \sqrt[4]{-16}$ $\varnothing$

g) $\sqrt[5]{x^5} = \sqrt[5]{32}$ $x = 2$

h) $\sqrt[5]{x^5} = \sqrt[5]{-32}$ $x = -2$

i) $\sqrt[6]{x^6} = \sqrt[6]{64}$ $x = \pm 2$

j) $\sqrt[6]{x^6} = \sqrt[6]{-64}$ $\varnothing$

k) $\sqrt[7]{x^7} = \sqrt[7]{128}$ $x = 2$

l) $\sqrt[7]{x^7} = \sqrt[7]{-128}$ $x = -2$

m) $\sqrt[8]{x^8} = \sqrt[8]{0}$ $x = 0$

n) $\sqrt[9]{x^9} = \sqrt[9]{0}$ $x = 0$

3. Solve for x.

a) $x^2 = 3$ $x = \pm\sqrt{3}$

b) $x^2 = -3$ $\varnothing$

c) $x^3 = 3$ $x = \sqrt[3]{3}$

d) $x^3 = -3$ $x = \sqrt[3]{-3}$

e) $x^4 = 3$ $x = \pm\sqrt[4]{3}$

f) $x^4 = -3$ $\varnothing$

g) $x^5 = 3$ $x = \sqrt[5]{3}$

h) $x^5 = -3$ $x = \sqrt[5]{-3}$

i) $x^6 = 3$ $x = \pm\sqrt[6]{3}$

j) $x^6 = -3$ $\varnothing$

k) $x^7 = 3$ $x = \sqrt[7]{3}$

l) $x^7 = -3$ $x = \sqrt[7]{-3}$

4. Simplify. Let the variables be any real numbers.

a) $\sqrt{9x^2} = \sqrt{9} \cdot \sqrt{x^2}$ $3|x|$

b) $\sqrt{x^5} = \sqrt{x^4} \cdot \sqrt{x}$ $x^2\sqrt{x}; x \geq 0$

c) $\sqrt{x^6y^4} = \sqrt{x^6} \cdot \sqrt{y^4}$ $|x^3|y^2$

d) $\sqrt{xy^4} = \sqrt{x} \cdot \sqrt{y^4}$ $y^2\sqrt{x}; x \geq 0$

e) $\sqrt{x^2y} = \sqrt{x^2} \cdot \sqrt{y}$ $|x|\sqrt{y}; y \geq 0$

f) $\sqrt{xy^2}$ $|y|\sqrt{x}; x \geq 0$

g) $\sqrt{x^2y^2}$ $|xy|$

h) $\sqrt{x^2y^4}$ $|x|y^2$

i) $\sqrt{x^4y^2}$ $x^2|y|$

j) $\sqrt{x^6y^4}$ $|x^3|y^2$

k) $\sqrt{x^5y^2}$ $x^2|y|\sqrt{x}; x \geq 0$

l) $\sqrt{x^{10}}$ $|x^5|$

m) $\sqrt{16x^6y^8}$ $4|x^3|y^4$

n) $\sqrt{4x^9}$ $2x^4\sqrt{x}; x \geq 0$

5. Simplify. Assume all variables represent positive numbers.

a) $\sqrt[3]{27}$ 3

b) $\sqrt[3]{-27}$ -3

c) $\sqrt[3]{x^3}$ x

d) $\sqrt[3]{-x^3}$ $-x$

e) $\sqrt[3]{x^6}$ x^2

f) $\sqrt[3]{-x^6}$ $-x^2$

g) $\sqrt[4]{16}$ 2

h) $\sqrt[4]{-16}$ $\emptyset$

i) $\sqrt[4]{x^4y^6} = \sqrt[4]{x^4} \cdot \sqrt[4]{y^4 \cdot y^2}$ $x\sqrt[4]{y^2}$

j) $\sqrt[4]{x^5y^7}$ $x\sqrt[4]{xy^3}$

$\sqrt[4]{x^4} \cdot \sqrt[4]{x} \cdot \sqrt[4]{y^4} \cdot \sqrt[4]{y^3}$

6. Simplify. Assume that the variables represent real numbers.

a) $\sqrt{(x-2)^2}$ $|x-2|$ b) $\sqrt[3]{(x-2)^3}$ $x-2$

c) $\sqrt[4]{(x-2)^4}$ $|x-2|$ d) $\sqrt[5]{(x-2)^5}$ $x-2$

e) $\sqrt{(2-x)^2}$ $|2-x|$ f) $\sqrt[3]{(2-x)^3}$ $2-x$

g) $\sqrt[4]{(2-x)^4}$ $|2-x|$ h) $\sqrt[5]{(2-x)^5}$ $2-x$

i) $\sqrt{x^2+4x+4}$ $(x+2)^2$ $|x+2|$ j) $\sqrt{x^2-2x+1}$ $(x-1)^2$ $|x-1|$

k) $\sqrt{(x^2+2x+1)(x^2-2x+1)}$ $(x+1)^2(x-1)^2$ $|x+1||x-1|$ l) $\sqrt[3]{(x-1)(x^2-2x+1)}$ $(x-1)^2$ $x-1$

m) $\sqrt[3]{\frac{-4}{27x^6}}$ $\frac{\sqrt[3]{-4}}{3x^2}$ n) $\sqrt{\frac{10}{4x^2}}$ $\frac{\sqrt{10}}{2|x|}$

o) $\sqrt[4]{\frac{16}{81x^8}}$ $\frac{2}{3x^2}$ p) $\sqrt[4]{\frac{81}{16x^{12}}}$ $\frac{3}{2|x^3|}$

7. Re-write as a single radical.

a) $\sqrt[3]{\sqrt[4]{x}}$ $\sqrt[12]{x}$ b) $\sqrt[2]{\sqrt[3]{x^2y}}$ $\sqrt[6]{x^2y}$

c) $\sqrt[4]{\sqrt[3]{\sqrt[2]{x}}}$ $\sqrt[24]{x}$ d) $\sqrt[2]{\sqrt[3]{xy^2z^3}}$ $\sqrt[6]{xy^2z^3}$

e) $\sqrt[2]{\sqrt[2]{\sqrt[2]{xy^2z^3}}}$ $\sqrt[8]{xy^2z^3}$ f) $\sqrt[4]{\sqrt[3]{\sqrt[2]{xy^2z^3}}}$ $\sqrt[24]{xy^2z^3}$

8. Explain why $\sqrt{x^2y} = |x| \cdot \sqrt{y},\ y \geq 0$ needs an absolute value.

x changed from an even → odd power

9. Explain why $\sqrt{x^4y} = x^2\sqrt{y},\ y \geq 0$ does not need an absolute value.

x stayed an even power

10. Explain why $\sqrt{x^3} = x\sqrt{x}$ requires the restriction $x \geq 0$.

x must be +ve under the radical

11. Explain why $\sqrt{-4}$ is not a real number, and why $\sqrt[3]{-8}$ is a real number.

$\sqrt{-4}$ is an even power ⇒ must have a + #

$\sqrt[3]{-8}$ is an odd power

$\therefore \sqrt[3]{-8} = -2$

12. What is wrong? $\sqrt{(-3)^2} = (\sqrt{-3})^2$

$\sqrt{9}$ ↑ undefined.

3

13. What is wrong? $\sqrt{x^2+y^2} = x + y$

doesn't simplify

14. What is wrong? $\sqrt[6]{(-8)^2} = \sqrt[3]{-8} = -2$

$\sqrt[6]{64} = 2$

15. What is wrong? $(x^3+y^3)^{\frac{1}{3}} = x + y$

$\sqrt[3]{x^3+y^3}$

doesn't simplify

16. What is wrong? $\frac{\sqrt[3]{27}}{\sqrt{9}} = \sqrt[3]{\frac{27}{9}} = \sqrt[3]{3}$

this is a square root!

17. What is wrong? $\sqrt{x^2+8x+16} = \sqrt{(x+4)^2} = |x+4|$

1.3 Exercise Set

1. Simplify.

a) $-\sqrt{9}$ -3

b) $\sqrt{\frac{1}{4}}$ $\frac{1}{2}$

c) $\sqrt{0.09}$ 0.3

d) $-\sqrt{100}$ -10

e) $\sqrt[3]{-8}$ -2

f) $\sqrt[3]{27}$ 3

g) $-\sqrt[4]{16}$ -2

h) $\sqrt[5]{-32}$ -2

i) $\sqrt[4]{-16}$ $\emptyset$

j) $-\sqrt[6]{64}$ -2

k) $\sqrt[4]{0.0016}$ 0.2

l) $\sqrt[6]{0.000064}$ 0.2

2. Solve for x.

a) $x^2 = 9$ $x = \pm 3$

b) $x^2 = \frac{1}{4}$ $x = \pm \frac{1}{2}$

c) $x^2 = 0.09$ $x = \pm 0.3$

d) $x^2 = 100$ $x = \pm 10$

e) $x^3 = -8$ $x = -2$

f) $x^3 = 27$ $x = 3$

g) $x^4 = 16$ $x = \pm 2$

h) $x^5 = -32$ $x = -2$

i) $x^4 = -16$ $\emptyset$

j) $x^6 = 64$ $x = \pm 2$

k) $\sqrt[4]{0.0016}$ $x = \pm 0.2$

l) $\sqrt[6]{0.000064}$ $x = \pm 0.2$

3. Change to simplest radical form.

a) $\sqrt{32} = \sqrt{16}\cdot\sqrt{2}$ $4\sqrt{2}$

b) $\sqrt{80} = \sqrt{16}\cdot\sqrt{5}$ $4\sqrt{5}$

c) $\sqrt{75} = \sqrt{25}\cdot\sqrt{3}$ $5\sqrt{3}$

d) $\sqrt[3]{-54} = \sqrt[3]{-27}\cdot\sqrt[3]{2}$ $-3\sqrt[3]{2}$

e) $\sqrt[3]{16} = \sqrt[3]{8}\cdot\sqrt{2}$ $2\sqrt{2}$

f) $\sqrt[3]{-72} = \sqrt[3]{-8}\cdot\sqrt[3]{9}$ $-2\sqrt[3]{9}$

g) $3\sqrt{45} = 3\sqrt{9}\cdot\sqrt{5}$ $9\sqrt{5}$

h) $\frac{3}{2}\sqrt{128} = \frac{3}{2}\sqrt{64}\cdot\sqrt{2}$ $12\sqrt{2}$

i) $\frac{4}{3}\sqrt{54} = \frac{4}{3}\sqrt{9}\cdot\sqrt{6}$ $4\sqrt{6}$

j) $-2\sqrt[3]{162} = -2\sqrt[3]{27\times 6}$ $-6\sqrt[3]{6}$

k) $\frac{5}{6}\sqrt[3]{-32} = \frac{5}{6}\sqrt[3]{-8}\cdot\sqrt[3]{4}$ $-\frac{5}{3}\sqrt[3]{4}$

l) $-\frac{5}{7}\sqrt[3]{686} = -\frac{5}{7}\sqrt[3]{343\times 2}$ $-5\sqrt[3]{2}$

m) $\sqrt[4]{162} = \sqrt[4]{81\cdot 2}$ $3\sqrt[4]{2}$

n) $\frac{2}{3}\sqrt[4]{1296} = \frac{2}{3}\times 6$ 4

4. Express in simplest radical form. All variables represent non-negative real numbers.

a) $\sqrt{x^3y^2}$ $xy\sqrt{x}$

b) $\sqrt{49x^3y^5}$ $7xy^2\sqrt{xy}$

c) $\sqrt{18x^6}$ $3x^3\sqrt{2}$

d) $\sqrt{25x^6y^{11}}$ $5x^3y^5\sqrt{y}$

e) $\sqrt{\frac{5}{x^4}}$ $\frac{\sqrt{5}}{x^2}$

f) $\sqrt{\frac{x^4y^5}{144}}$ $\frac{x^2y^2\sqrt{y}}{12}$

g) $\sqrt[3]{40x^3}$ $2x\sqrt[3]{5}$

h) $\sqrt[3]{-8x^9}$ $-2x^3$

i) $\sqrt[3]{-64x^3y^6}$ $-4xy^2$

j) $\sqrt[3]{216x^5}$ $6x\sqrt[3]{x^2}$

k) $\sqrt[4]{16x^4y^8}$ $2xy^2$

l) $\sqrt[4]{162x^5y^6} = \sqrt[4]{81\cdot 2x^5y^6}$ $3xy\sqrt[4]{2xy^2}$

m) $\sqrt[5]{-32x^4}$ $-2\sqrt[5]{x^4}$

n) $\sqrt[6]{128x^6y^{12}} = \sqrt[6]{64\cdot 2x^6y^{12}}$ $2xy^2\sqrt[6]{2}$

5. Write as an entire radical.

a) $2\sqrt{3} = \sqrt{2 \cdot 2 \cdot 3}$ $\sqrt{12}$

b) $-4\sqrt{5} = -\sqrt{4 \cdot 4 \cdot 5}$ $-\sqrt{80}$

c) $3\sqrt{4} = \sqrt{3 \cdot 3 \cdot 4}$ $\sqrt{36}$

d) $\frac{2}{3}\sqrt{5} = \sqrt{\frac{2}{3} \cdot \frac{2}{3} \cdot 5}$ $\sqrt{20/9}$

e) $2\sqrt[3]{3} = \sqrt[3]{2 \cdot 2 \cdot 2 \cdot 3}$ $\sqrt[3]{24}$

f) $-4\sqrt[3]{5} = -\sqrt[3]{4 \cdot 4 \cdot 4 \cdot 5}$ $-\sqrt[3]{320}$

g) $3\sqrt[3]{4} = \sqrt[3]{3 \cdot 3 \cdot 3 \cdot 4}$ $\sqrt[3]{108}$

h) $\frac{2}{3}\sqrt[3]{5} = \sqrt[3]{\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot 5}$ $\sqrt[3]{40/27}$

i) $2\sqrt[4]{3} = \sqrt[4]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$ $\sqrt[4]{48}$

j) $-4\sqrt[4]{5} = -\sqrt[4]{4 \cdot 4 \cdot 4 \cdot 4 \cdot 5}$ $-\sqrt[4]{1280}$

k) $3\sqrt[4]{4} = \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3 \cdot 4}$ $\sqrt[4]{324}$

l) $\frac{2}{3}\sqrt[4]{5} = \sqrt[4]{\frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot 5}$ $\sqrt[4]{80/81}$

m) $2\sqrt[5]{3} = \sqrt[5]{2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3}$ $\sqrt[5]{96}$

n) $2\sqrt[6]{3} = \sqrt[6]{2^6 \cdot 3}$ $\sqrt[6]{192}$

6. Without using a calculator, determine which value is greater using $>$ or $<$.

a) $\sqrt{2} > \sqrt[3]{2}$

b) $\sqrt{\frac{1}{2}} < \sqrt[3]{\frac{1}{2}}$

c) $\sqrt{\frac{8}{9}} > \sqrt{\frac{7}{8}}$

d) $\sqrt{\frac{9}{8}} < \sqrt{\frac{8}{7}}$

e) $\sqrt[3]{\frac{2}{3}} < \sqrt[3]{\frac{3}{4}}$

f) $\sqrt[3]{\frac{3}{2}} > \sqrt[3]{\frac{4}{3}}$

g) $2\sqrt{14} < 3\sqrt{7}$

h) $-5\sqrt{3} < -6\sqrt{2}$

i) $2\sqrt[3]{10} < 3\sqrt[3]{3}$

j) $-2\sqrt[4]{10} > -3\sqrt[4]{2}$

k) $5\sqrt{4} > 3\sqrt{11}$

l) $-5\sqrt{4} < -3\sqrt{11}$

7. Write as an entire radical. All variables represent non-negative real numbers.

a) $x\sqrt{y}$ $\sqrt{x^2y}$

b) $-9xy^2\sqrt{x}$ $-\sqrt{81x^3y^4}$

c) $2x\sqrt[3]{5xy}$ $\sqrt[3]{40x^4y}$

d) $-3xy^2\sqrt[3]{3}$ $-\sqrt[3]{81x^3y^6}$

e) $-xy\sqrt[4]{xy^3}$ $-\sqrt[4]{x^5y^7}$

f) $3xy\sqrt[4]{2y^3}$ $\sqrt[4]{162x^4y^7}$

g) $\dfrac{x^2\sqrt[4]{xy}}{y}$ $\sqrt[4]{x^9/y^3}$

h) $\dfrac{-\sqrt[3]{7}}{3x^2}$ $-\sqrt[3]{7/27x^6}$

i) $\dfrac{\sqrt[3]{20x^2}}{4x}$ $\sqrt[3]{20/64x} = \sqrt[3]{5/16x}$

j) $\dfrac{\sqrt[4]{8x}}{2x}$ $\sqrt[4]{8x/16x^4} = \sqrt[4]{1/2x^3}$

k) $\dfrac{\sqrt[5]{24}}{2} = \sqrt[5]{24/32} = \sqrt[5]{3/4}$

l) $\dfrac{\sqrt[6]{8x}}{2x^2}$ $\sqrt[6]{8x/64x^{12}} = \sqrt[6]{1/8x^{11}}$

8. Express each radical in simplest radical form. All variables represent non-negative real numbers.

a) $(\sqrt{3a^2b})(\sqrt{6ab^5})$

$= \sqrt{18a^3b^6}$

$= 3ab^3\sqrt{2a}$

b) $(4x\sqrt{10xy})(3y\sqrt{2x})$

$= 12xy\sqrt{20x^2y}$

$= 24x^2\sqrt{5y}$

c) $(2x\sqrt[3]{2y^4})(x^2\sqrt[3]{4y^2})$

$= 2x^3\sqrt[3]{8y^6}$

$= 4x^3y^2$

d) $(ab\sqrt[3]{2ab^2})(3a\sqrt[3]{4a^2b^2})$

$= 3a^2b\sqrt[3]{8a^3b^4}$

$= 6a^3b^2\sqrt[3]{b}$

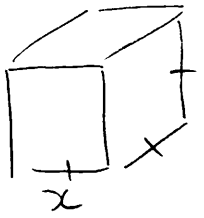
e) $\dfrac{9x^2\sqrt{x^2y^5}}{3x^5\sqrt{x^6y}} = \sqrt[3]{\dfrac{y^4}{x^4}}$

$= \dfrac{3}{x^3}\cdot\dfrac{y^2}{x^2} = \dfrac{3y^2}{x^5}$

f) $\dfrac{\sqrt[3]{81x^2y^5}}{\sqrt[3]{x^5y}} = \sqrt[3]{\dfrac{81y^4}{x^3}}$

$= \dfrac{3y}{x}\sqrt[3]{3y}$

9. A cube-shaped box has a volume of 128 cm³. What is the length of each side of the box?



$x^3 = 128\text{cm}^3$

$x = \sqrt[3]{128}$

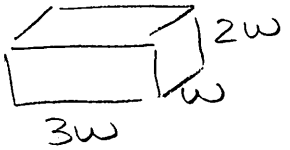
$x = 4\sqrt{2}\text{ cm}$

10. The volume of a sphere has the formula $V = \frac{4}{3}\pi r^3$. If the volume is multiplied by 8, how is the radius affected?

$8 \cdot V = \frac{4}{3}\pi r^3 \cdot 8$

radius is doubled.

11. A rectangular solid has a volume of 192 cm³. If the height is twice the width and the length is three times the width, what are the dimensions of the rectangular solid?



$V = 192\text{cm}^3$

$V = 6w^3 = 192$

$w^3 = 32$

$w = \sqrt[3]{32}$

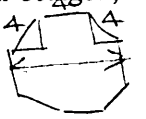
$w = 2\sqrt[3]{4}$

$l = 6\sqrt[3]{4}\text{ cm}$

$w = 2\sqrt[3]{4}\text{ cm}$

$h = 4\sqrt[3]{4}\text{ cm}$

12. Find the distance between the parallel sides of a regular octagon, with sides of length 4 cm.

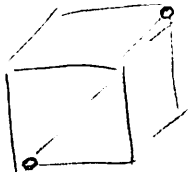


$= 2\sqrt{2}$

distance $= 2(2\sqrt{2}) + 4\text{cm}$

$= 4\sqrt{2} + 4\text{cm}$

13. What is the length of the diagonal that connects two opposite corners of a cube with sides 4 cm?



$\sqrt{4^2 + 4^2 + 4^2}$

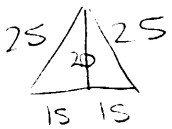
$= \sqrt{48} = 4\sqrt{3}\text{cm}$

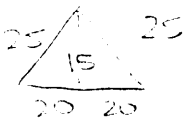
14. What is the length of the diagonal of a rectangular solid, with lengths 3 cm, 4 cm, and 5 cm?

$\sqrt{3^2 + 4^2 + 5^2} = \sqrt{50}$

$= 5\sqrt{2}\text{cm}$

15. One isosceles triangle has sides 25 cm, 25 cm, and 30 cm. The other isosceles triangle has sides 25 cm, 25 cm, and 40 cm. Which triangle has the greatest area?

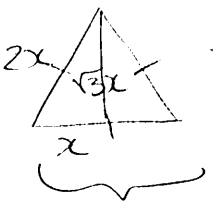




$A = \frac{1}{2}(30)(20)$ $A = \frac{1}{2}(40)(15)$

$A = 300\text{cm}^2$ same!

16. The area of an equilateral triangle is $50\sqrt{3}$ cm². Find the length of the sides.



length $= 2x$

$\sqrt{3}x \cdot x = 50\sqrt{3}$

$x^2 = 50$

$x = \sqrt{50}$

$x = 5\sqrt{2}\text{cm}$

$2x = 10\sqrt{2}\text{cm}$

1.4 Exercise Set

1. Fill in the blanks to make the statement true.

   a) Like terms have the same radicand and index.

   b) Like radicals have the same variable and exponent.

   c) The radicals $\sqrt{12}$ and $\sqrt{27}$ can be simplified so they are like radicals.

2. Indicate whether the expressions are like radicals.

   a) $5\sqrt{3}$ and $3\sqrt{5}$ y (n)

   b) $3\sqrt{2x}$ and $2\sqrt{2x}$ (y) n

   c) $3\sqrt{x}$ and $3\sqrt{y}$ y (n)

   d) $5\sqrt{x^2y}$ and $2\sqrt{xy^2}$ y (n)

   e) $12\sqrt{2x}$ and $6\sqrt[3]{2x}$ y (n)

   f) $-5\sqrt[4]{3x}$ and $6\sqrt[4]{3x}$ (y) n

3. Explain what is wrong with each solution.

   a) $6\sqrt{7} - 4\sqrt{2} = 2\sqrt{5}$

   b) $6\sqrt{5} + 3\sqrt{6} = 9\sqrt{11}$

   c) $8 - 5\sqrt{2} = 3\sqrt{2}$
   ↑ not radical

   d) $6 + 2\sqrt{3} = 8\sqrt{3}$
   ↑ not radical

   e) $\sqrt{x} + \sqrt{y} = \sqrt{xy}$

   f) $2x\sqrt{y} + 3y\sqrt{x} = 5xy\sqrt{xy}$

4. Complete each solution.

   a) $2\sqrt{40} + 7\sqrt{90} = 2\sqrt{4 \cdot 10} + 7\sqrt{9 \cdot 10}$
   $= 2 \cdot 2\sqrt{10} + 7 \cdot 3\sqrt{10}$
   $= 4\sqrt{10} + 21\sqrt{10}$
   $= 25\sqrt{10}$

   b) $5\sqrt{20} - 2\sqrt{125} = 5\sqrt{4 \cdot 5} - 2\sqrt{25 \cdot 5}$
   $= 5 \cdot 2\sqrt{5} - 2 \cdot 5\sqrt{5}$
   $= 10\sqrt{5} + -10\sqrt{5}$
   $= 0$

5. Simplify each expression.

a) $6\sqrt{2}+3\sqrt{2}$ $=9\sqrt{2}$

b) $5\sqrt{3}-9\sqrt{3}$ $=-4\sqrt{3}$

c) $7\sqrt{32}+4\sqrt{2}$

$7\cdot 4\sqrt{2}+4\sqrt{2}$

$28\sqrt{2}+4\sqrt{2}=32\sqrt{2}$

d) $7\sqrt{48}+4\sqrt{3}$

$7\cdot 4\sqrt{3}+4\sqrt{3}$

$28\sqrt{3}+4\sqrt{3}=32\sqrt{3}$

e) $3\sqrt[3]{54}+5\sqrt[3]{16}$

$3\cdot 3\sqrt[3]{2}+5\cdot 2\sqrt[3]{2}$

$9\sqrt[3]{2}+10\sqrt[3]{2}=19\sqrt[3]{2}$

f) $5\sqrt[4]{48}-2\sqrt[4]{243}$

$5\cdot 2\sqrt[4]{3}-2\cdot 3\sqrt[4]{3}$

$10\sqrt[4]{3}-6\sqrt[4]{3}=4\sqrt[4]{3}$

g) $7\sqrt{63}-2\sqrt{28}$

$7\cdot 3\sqrt{7}-2\cdot 2\sqrt{7}$

$21\sqrt{7}-4\sqrt{7}=17\sqrt{7}$

h) $3\sqrt{40}-8\sqrt{90}$

$3\cdot 2\sqrt{10}-8\cdot 3\sqrt{10}$

$6\sqrt{10}-24\sqrt{10}=-18\sqrt{10}$

i) $4\sqrt{12}+2\sqrt{27}-3\sqrt{75}$

$4\cdot 2\sqrt{3}+2\cdot 3\sqrt{3}-3\cdot 5\sqrt{3}$

$8\sqrt{3}+6\sqrt{3}-15\sqrt{3}=-\sqrt{3}$

j) $5\sqrt{18}-4\sqrt{50}-2\sqrt{72}$

$5\cdot 3\sqrt{2}-4\cdot 5\sqrt{2}-2\cdot 6\sqrt{2}$

$15\sqrt{2}-20\sqrt{2}-12\sqrt{2}$

$=-17\sqrt{2}$

k) $4\sqrt{50}+3\sqrt{12}-5\sqrt{27}$

$4\cdot 5\sqrt{2}+3\cdot 2\sqrt{3}-5\cdot 3\sqrt{3}$

$20\sqrt{2}+6\sqrt{3}-15\sqrt{3}=$

$20\sqrt{2}-9\sqrt{3}$

l) $5\sqrt{72}+3\sqrt{48}-4\sqrt{128}$

$5\cdot 6\sqrt{2}+3\cdot 4\sqrt{3}-4\cdot 8\sqrt{2}$

$30\sqrt{2}+12\sqrt{3}-32\sqrt{2}$

$12\sqrt{3}-2\sqrt{2}$

m) $\sqrt{162}-\sqrt{50}+\sqrt{75}-\sqrt{108}$

$9\sqrt{2}-5\sqrt{2}+5\sqrt{3}-6\sqrt{3}$

$4\sqrt{2}-\sqrt{3}$

n) $\sqrt{48}+\sqrt{8}-\sqrt{27}-\sqrt{32}$

$4\sqrt{3}+2\sqrt{2}-3\sqrt{3}-4\sqrt{2}$

$\sqrt{3}-2\sqrt{2}$

o) $\frac{1}{4}\sqrt{80}+\frac{2}{3}\sqrt{45}-\frac{1}{2}\sqrt{20}$

$\frac{1}{4}\cdot 4\sqrt{5}+\frac{2}{3}\cdot 3\sqrt{5}-\frac{1}{2}\cdot 2\sqrt{5}$

$\sqrt{5}+2\sqrt{5}-\sqrt{5}=2\sqrt{5}$

p) $\frac{2}{3}\sqrt{12}-\frac{5}{2}\sqrt{48}+\frac{1}{4}\sqrt{108}$

$\frac{2}{3}\cdot 2\sqrt{3}-\frac{5}{2}\cdot 4\sqrt{3}+\frac{1}{4}\cdot 6\sqrt{3}$

$\frac{4}{3}\sqrt{3}-10\sqrt{3}+\frac{3}{2}\sqrt{3}=\frac{8}{6}\sqrt{3}-\frac{60}{6}\sqrt{3}+\frac{9}{6}\sqrt{3}$

$=-\frac{43}{6}\sqrt{3}$

q) $5\sqrt[4]{32}+2\sqrt[4]{8}\cdot\sqrt[4]{4}$

$5\cdot 2\sqrt[4]{2}+2\sqrt[4]{32}$

$10\sqrt[4]{2}+2\cdot 2\sqrt[4]{2}$

$10\sqrt[4]{2}+4\sqrt[4]{2}=14\sqrt[4]{2}$

r) $8\sqrt[3]{16}-10\sqrt[3]{3}\cdot\sqrt[3]{18}$

$8\cdot 2\sqrt[3]{2}-10\sqrt[3]{54}$

$16\sqrt[3]{2}-10\cdot 3\sqrt[3]{2}$

$16\sqrt[3]{2}-30\sqrt[3]{2}=-14\sqrt[3]{2}$

6. Simplify each expression. All variables represent positive numbers.

a) $\sqrt{3x^2} + \sqrt{12x^2}$

$x\sqrt{3} + 2x\sqrt{3}$

$3x\sqrt{3}$

b) $3\sqrt{2x^3} + 5x\sqrt{8x}$

$3x\sqrt{2x} + 5x \cdot 2\sqrt{2x}$

$13x\sqrt{2x}$

c) $\sqrt{4x^7} - 5x^2\sqrt{x^3} + 3x\sqrt{x^5}$

$2x^3\sqrt{x} - 5x^2 \cdot x\sqrt{x} + 3x \cdot x^2\sqrt{x}$

$= 0$

d) $\sqrt{9x^3} - \sqrt{25x^3} + x\sqrt{16x}$

$3x\sqrt{x} - 5x\sqrt{x} + 4x\sqrt{x}$

$2x\sqrt{x}$

e) $3\sqrt{125x^2y} + 6x\sqrt{80y}$

$3 \cdot 5x\sqrt{5y} + 6x \cdot 4\sqrt{5y}$

$39x\sqrt{5y}$

f) $5\sqrt{12x} - 3\sqrt{27x}$

$5 \cdot 2\sqrt{3x} - 3 \cdot 3\sqrt{3x}$

$\sqrt{3x}$

g) $5x\sqrt{63y} + 3\sqrt{28x^2y}$

$5x \cdot 3\sqrt{7y} + 3 \cdot 2x\sqrt{7y}$

$21x\sqrt{7y}$

h) $3y\sqrt{24x^2y^2} + 6x\sqrt{54y^3}$

$3y \cdot 2xy\sqrt{6} + 6x \cdot 3y\sqrt{6y}$

$6xy^2\sqrt{6} + 18xy\sqrt{6y}$

i) $\sqrt{\frac{28}{x^2}} + \sqrt{\frac{7}{4x^2}}$

$\left(\frac{2}{2}\right)\frac{2\sqrt{7}}{x} + \frac{1\sqrt{7}}{2x} = \frac{5\sqrt{7}}{2x}$

j) $\frac{\sqrt{99}}{5x} + \sqrt{\frac{44}{x^2}}$

$\frac{3\sqrt{11}}{5x} + \frac{2\sqrt{11}}{x}\left(\frac{5}{5}\right) = \frac{13\sqrt{11}}{5x}$

k) $5\sqrt{8x^2y^3} - 3x\sqrt{32y^3}$

$5 \cdot 2xy\sqrt{2y} - 3x \cdot 4y\sqrt{2y}$

$-2xy\sqrt{2y}$

l) $\sqrt{9y+27} + \sqrt{y+3}$

$\sqrt{9(y+3)} + \sqrt{y+3}$

$3\sqrt{y+3} + \sqrt{y+3} = 4\sqrt{y+3}$

m) $\sqrt{9x-9} + \sqrt{x-1}$

$\sqrt{9(x-1)} + \sqrt{x-1}$

$3\sqrt{x-1} + \sqrt{x-1} = 4\sqrt{x-1}$

n) $2x\sqrt{4x^2z} + 5x\sqrt{9z} - 10x\sqrt{25z}$

$2x \cdot 2x\sqrt{z} + 5x \cdot 3\sqrt{z} - 10x \cdot 5\sqrt{z}$

$4x^2\sqrt{z} - 35x\sqrt{z}$

o) $\frac{1}{3}\sqrt{x^2} - \frac{5}{3}\sqrt{x^2} + \frac{5}{6}\sqrt{x^2}$

$\left(\frac{2}{2}\right)\frac{1}{3}x - \frac{5}{3}x + \frac{5}{6}x = -\frac{3}{6}x = -\frac{1}{2}x$

p) $\frac{2}{3}\sqrt{9x^2} + \frac{5}{3}\sqrt{4x^2} + \sqrt{25x^2}$

$\frac{2}{3} \cdot 3x + \frac{5}{3} \cdot 2x + 5x\left(\frac{3}{3}\right) = \frac{31x}{3}$

q) $\sqrt{x^2-6x+9} - \sqrt{x^2-2x+1}$

$\sqrt{(x-3)^2} - \sqrt{(x-1)^2}$

$x - 3 - (x-1)$

$= -2$

r) $\sqrt{x^2+4x+4} - \sqrt{x^2+10x+25}$

$\sqrt{(x+2)^2} - \sqrt{(x+5)^2}$

$x + 2 - (x+5)$

$= -3$

7. Simplify each expression. All variables represent positive numbers.

a) $\sqrt[3]{27} - 4\sqrt[3]{8}$

$3 - 4 \cdot 2$

$= -5$

b) $3\sqrt[3]{6} + 2\sqrt[3]{48}$

$3\sqrt[3]{6} + 2 \cdot 2\sqrt[3]{6}$

$= 7\sqrt[3]{6}$

c) $\sqrt[3]{x^4} - \sqrt[3]{x^7}$

$x\sqrt[3]{x} - x^2\sqrt[3]{x}$

d) $x\sqrt[3]{8x^5} + \sqrt[3]{27x^8}$

$x \cdot 2x\sqrt[3]{x^2} + 3x^2\sqrt[3]{x^2}$

$= 5x^2\sqrt[3]{x^2}$

e) $6\sqrt[3]{8x^2} - 2\sqrt[3]{27x^2}$

$6 \cdot 2\sqrt[3]{x^2} - 2 \cdot 3\sqrt[3]{x^2}$

$= 6\sqrt[3]{x^2}$

f) $4\sqrt[3]{27x^2} + 6\sqrt[3]{8x^2}$

$4 \cdot 3\sqrt[3]{x^2} + 6 \cdot 2\sqrt[3]{x^2}$

$= 24\sqrt[3]{x^2}$

g) $3\sqrt[4]{x^5} - 2x\sqrt[4]{16x}$

$3x\sqrt[4]{x} - 2x(2)\sqrt[4]{x}$

$= -x\sqrt[4]{x}$

h) $2\sqrt[4]{6x^7} - x\sqrt[4]{96x^3}$

$2x\sqrt[4]{6x^3} - x(2)\sqrt[4]{6x^3}$

$= 0$

i) $-4\sqrt[3]{256x^4} - 2x\sqrt[3]{32x}$

$-4(4x)\sqrt[3]{4x} - 2x(2)\sqrt[3]{4x}$

$= -20x\sqrt[3]{4x}$

j) $\frac{5}{2}\sqrt[3]{16x^4y^5} + xy\sqrt[3]{54xy^2}$

$\frac{5}{2}(2xy)\sqrt[3]{2xy^2} + xy(3)\sqrt[3]{2xy^2}$

$= 8xy\sqrt[3]{2xy^2}$

k) $-\frac{\sqrt[3]{2x^4}}{9} + \sqrt[3]{\frac{250x^4}{27}}$

$-\frac{x}{9}\sqrt[3]{2x} + \frac{5x}{3}\sqrt[3]{2x}$

$= \frac{14x}{9}\sqrt[3]{2x}$

l) $\frac{\sqrt[3]{x^5}}{8} + \frac{5x\sqrt[3]{x^2}}{4}$

$\frac{x\sqrt[3]{x^2}}{8} + \frac{5x\sqrt[3]{x^2}}{4} = \frac{11x\sqrt[3]{x^2}}{8}$

m) $3\sqrt[4]{16x} - 5\sqrt[4]{x^5} + x\sqrt[4]{81x}$

$3(2)\sqrt[4]{x} - 5x\sqrt[4]{x} + 3x\sqrt[4]{x}$

$= 6\sqrt[4]{x} + 2x\sqrt[4]{x}$

n) $4\sqrt[4]{32x^5} - 2x\sqrt[4]{2x} + 7\sqrt[4]{x^5}$

$4(2x)\sqrt[4]{2x} - 2x\sqrt[4]{2x} + 7x\sqrt[4]{x}$

$= 6x\sqrt[4]{2x} + 7x\sqrt[4]{x}$

o) $\sqrt{20x^3y} - \sqrt{45x^5y^3} + \sqrt{80x^7y^5}$

$2x\sqrt{5xy} - 3x^2y\sqrt{5xy} + 4x^3y^2\sqrt{5xy}$

p) $x\sqrt{48xy^2} + y\sqrt{27x^3} - \sqrt{75x^3y^2}$

$x(4)\sqrt{3x} + y(3x)\sqrt{3x} - 5xy\sqrt{3x}$

$= 2xy\sqrt{3x}$

q) $\sqrt{8x^2y^3} + xy\sqrt[4]{64y^2}$

$2xy\sqrt{2y} + xy(2)\sqrt{4y^2}$

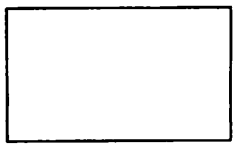
r) $\sqrt[3]{8x^3y} + \sqrt[6]{64x^6y^2}$

$2x\sqrt[3]{y} + 2x\sqrt[6]{y^2} = 2x\sqrt[3]{y} + 2x\sqrt[3]{y}$

$= 4x\sqrt[3]{y}$

8. Find the perimeter.

a) $2\sqrt{12}$ cm



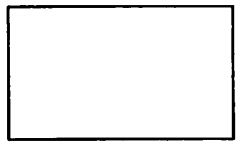
$2(2\sqrt{12}) + 2(3\sqrt{8})$

$= 4\sqrt{12} + 6\sqrt{8}$

$= 4(2)\sqrt{3} + 6(2)\sqrt{2}$

$= 8\sqrt{3} + 12\sqrt{2}$ cm

b) $3\sqrt{20}$ cm



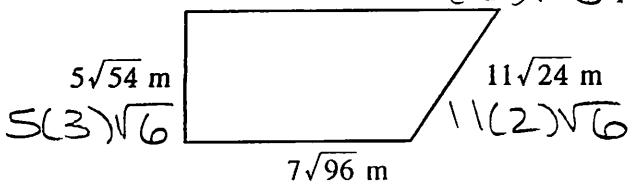
$2(3\sqrt{20}) + 2(\sqrt{125})$

$= 6\sqrt{20} + 2\sqrt{125}$

$= 6(2)\sqrt{5} + 2(5)\sqrt{5}$

$= 22\sqrt{5}$ cm

c) $8\sqrt{150}$ m $\quad 8(5)\sqrt{6}$.

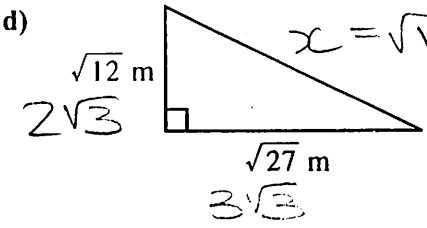


$5(3)\sqrt{6}$ $\quad 11(2)\sqrt{6}$

$7(4)\sqrt{6}$

$105\sqrt{6}$ m

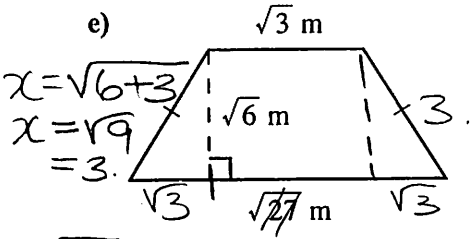
d)



$x = \sqrt{12 + 27} = \sqrt{39}$

$5\sqrt{3} + 39$ m

e)



$x = \sqrt{6+3}$

$x = \sqrt{9}$

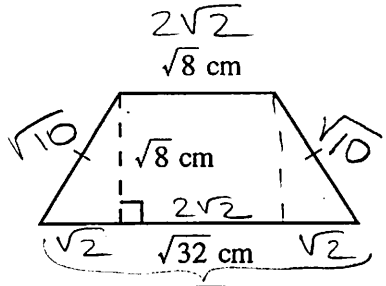
$= 3.$

$\sqrt{27} - \sqrt{3} = 3\sqrt{3} - \sqrt{3} = 2\sqrt{3}$

$3 + \sqrt{3} + 3 + 3\sqrt{3}$

$6 + 4\sqrt{3}$ m

f)



$4\sqrt{2}$

$2\sqrt{2} + 4\sqrt{2} + 2\sqrt{10}$

$6\sqrt{2} + 2\sqrt{10}$ cm

9. Explain why $\sqrt{a+b} \neq \sqrt{a} + \sqrt{b}$. Give examples.

$\sqrt{36+64} = \sqrt{100} = 10$

$\sqrt{36} + \sqrt{64} = 6 + 8 = 14$

1.5 Exercise Set

1. Fill in the blank to make the statement correct.

a) Changing a radical in the denominator of a fraction to a rational number is called rationalizing the denominator

b) In the fraction $\frac{3}{\sqrt{2}}$, the 3 is called the numerator and the $\sqrt{2}$ is called the denominator

c) $2+\sqrt{5}$ is the conjugate of $2-\sqrt{5}$.

d) To rationalize the denominator of $\frac{1}{\sqrt{x}}$, the numerator and denominator ~~are~~ multiplied by $\sqrt{x}$.

e) To rationalize the denominator of $\frac{1}{\sqrt{2}+1}$, the numerator and denominator ~~are~~ multiplied by $\sqrt{2}-1$.

2. Multiply. All variables represent positive numbers. Express answers in simplest radical form.

a) $(\sqrt{5})^2$ 5

b) $(-5\sqrt{2})^2 = 25(2)$ 50

c) $\sqrt{2}\cdot\sqrt{8} = \sqrt{16}$ 4

d) $\sqrt{3}\cdot\sqrt{27} = \sqrt{81}$ 9

e) $(-\sqrt[3]{5})^3$ -5

f) $\sqrt{6}\cdot\sqrt{8} = \sqrt{48}$ $4\sqrt{3}$

g) $\sqrt{a^4}\cdot\sqrt{a^6} = \sqrt{a^{10}}$ a^5

h) $(4\sqrt{x})(-3\sqrt{x})$ $-12x$

i) $\sqrt{2x}\cdot\sqrt{6x^3} = \sqrt{12x^4}$ $2x^2\sqrt{3}$

j) $(-3\sqrt{6})(4\sqrt{3}) = -12\sqrt{18}$ $= -12(3)\sqrt{2}$ $-36\sqrt{2}$

k) $-3\sqrt{2x}\cdot\sqrt{12y} = -3\sqrt{24xy}$ $= -3(2)\sqrt{6xy}$ $-6\sqrt{6xy}$

l) $(-2\sqrt[3]{4x^2})(-4\sqrt[3]{6x^2})$ $= 8\sqrt[3]{24x^4}$ $= 8(2x)\sqrt[3]{3x}$ $16x\sqrt[3]{3x}$

m) $(-2x\sqrt{xy^2})(6y\sqrt{x^2y})$ $-12x^2y^2\sqrt{xy}$ $-12xy\sqrt{x^3y^3}$ $-12xy(xy)\sqrt{xy}$

n) $(4xy\sqrt[3]{x^2y})(3xy\sqrt[3]{xy^2})$ $12x^2y^2\sqrt[3]{x^3y^3}$ $12x^3y^3$

3. Find each product and simplify.

a) $\sqrt{5}(\sqrt{8}+\sqrt{32})$

$\sqrt{40}+\sqrt{160}$

$2\sqrt{10}+4\sqrt{10} = 6\sqrt{10}$

b) $2\sqrt{3}(7\sqrt{5}-5\sqrt{3})$

$14\sqrt{15}-10\sqrt{9}$

$14\sqrt{15}-30$

c) $2\sqrt[3]{5}(3\sqrt[3]{3}-\sqrt[3]{25})$

$6\sqrt[3]{15}-2\sqrt[3]{125}$

$6\sqrt[3]{15}-10$

d) $2\sqrt[3]{2}(5\sqrt[3]{4}+3\sqrt[3]{2})$

$10\sqrt[3]{8}+6\sqrt[3]{4}$

$20+6\sqrt[3]{4}$

e) $(1-\sqrt{2})(1-\sqrt{2})$

$1-\sqrt{2}-\sqrt{2}+\sqrt{4}$

$1-2\sqrt{2}+2 = 3-2\sqrt{2}$

f) $(1-\sqrt{2})(1+\sqrt{2})$

$1-\sqrt{2}+\sqrt{2}-\sqrt{4}$

$1-2 = -1$

g) $(4+\sqrt{3})(4+\sqrt{3})$

$16+4\sqrt{3}+4\sqrt{3}+\sqrt{9}$

$19+8\sqrt{3}$

h) $(4+\sqrt{3})(4-\sqrt{3})$

$16-4\sqrt{3}+4\sqrt{3}-\sqrt{9}$

13

i) $(\sqrt{5}-\sqrt{2})(\sqrt{5}-\sqrt{2})$

$\sqrt{25}-\sqrt{10}-\sqrt{10}+\sqrt{4}$

$5-2\sqrt{10}+2 = 7-2\sqrt{10}$

j) $(\sqrt{5}+\sqrt{2})(\sqrt{5}+\sqrt{2})$

$\sqrt{25}+\sqrt{10}+\sqrt{10}+\sqrt{4}$

$5+2\sqrt{10}+2 = 7+2\sqrt{10}$

k) $(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})$

$\sqrt{25}+\sqrt{10}-\sqrt{10}-\sqrt{4}$

$5-2 = 3$

l) $\left(\frac{5-\sqrt{50}}{5}\right)\left(\frac{5-\sqrt{50}}{5}\right)$

$\frac{25-5\sqrt{50}-5\sqrt{50}+\sqrt{2500}}{25}$

$\frac{25-10\sqrt{50}+50}{25}$

$\frac{75-50\sqrt{2}}{25}$

$3-2\sqrt{2}$

m) $\left(\frac{5-\sqrt{50}}{5}\right)\left(\frac{5+\sqrt{50}}{5}\right)$

$\frac{25-5\sqrt{50}+5\sqrt{50}-50}{25} = \frac{-25}{25}$

$= -1$

n) $(\sqrt[3]{4}+2)(\sqrt[3]{2}-1)$

$\sqrt[3]{8}-\sqrt[3]{4}+2\sqrt[3]{2}-2$

$-\sqrt[3]{4}+2\sqrt[3]{2}$

o) $(\sqrt[4]{4}-1)(\sqrt[4]{4}+1)$

$\sqrt[4]{16}+\sqrt[4]{4}-\sqrt[4]{4}-1$

$2-1 = 1$

p) $(\sqrt[4]{4}-1)(\sqrt[4]{4}-1)$

$\sqrt[4]{16}-\sqrt[4]{4}-\sqrt[4]{4}+1$

$2-2\sqrt[4]{4}+1 = 3-2\sqrt[4]{4}$

q) $(\sqrt[3]{4}-1)(\sqrt[3]{16}+\sqrt[3]{4}+1)$

$\sqrt[3]{64}+\sqrt[3]{16}+\sqrt[3]{4}-\sqrt[3]{16}-\sqrt[3]{4}-1$

$4-1 = 3$

r) $(\sqrt[3]{2}+\sqrt[3]{3})(\sqrt[3]{4}-\sqrt[3]{6}+\sqrt[3]{9})$

$\sqrt[3]{8}-\sqrt[3]{12}+\sqrt[3]{18}+\sqrt[3]{12}-\sqrt[3]{18}+\sqrt[3]{27}$

$2+3=5$

4. Find each product and simplify. Assume all variables are positive.

a) $\sqrt{2x}(\sqrt{2}-\sqrt{x})$

$\sqrt{4x}-\sqrt{2x^2}$

$2\sqrt{x}-x\sqrt{2}$

b) $\sqrt{7y}(\sqrt{y}+\sqrt{7})$

$\sqrt{7y^2}+\sqrt{49y}$

$y\sqrt{7}+7\sqrt{y}$

c) $(2x-\sqrt{3})(2x+\sqrt{3})$

$4x^2+2x\sqrt{3}-2x\sqrt{3}-\sqrt{9}$

$4x^2-3$

d) $(2x-\sqrt{3})(2x-\sqrt{3})$

$4x^2-2x\sqrt{3}-2x\sqrt{3}+\sqrt{9}$

$4x^2-4x\sqrt{3}+3$

e) $(\sqrt{x+2})^2$

$x+2$

f) $(\sqrt{x}+2)(\sqrt{x}+2)$

$x+2\sqrt{x}+2\sqrt{x}+4$

$x+4\sqrt{x}+4$

g) $(\sqrt{x-3}-4)(\sqrt{x-3}-4)$

$x-3-4\sqrt{x-3}-4\sqrt{x-3}+16$

$x+13-8\sqrt{x-3}$

h) $(\sqrt{x-3}-4)(\sqrt{x-3}+4)$

$x-3+4\sqrt{x-3}-4\sqrt{x-3}-16$

$x-19$

i) $(3\sqrt{x}+\sqrt{y})(3\sqrt{x}+\sqrt{y})$

$9x+3\sqrt{xy}+3\sqrt{xy}+y$

$9x+6\sqrt{xy}+y$

j) $(\sqrt{x}+3\sqrt{6})(\sqrt{x}-3\sqrt{6})$

$x-3\sqrt{6x}+3\sqrt{6x}-9(6)$

$x-54$

k) $(\sqrt{x}-2)^2-(\sqrt{x}+2)^2$

$(\sqrt{x}-2)(\sqrt{x}-2)-(\sqrt{x}+2)(\sqrt{x}+2)$

$x-2\sqrt{x}-2\sqrt{x}+4-x-2\sqrt{x}-2\sqrt{x}-4$

$-8\sqrt{x}$

l) $(\sqrt{x}-y)(\sqrt{x}+y)$

$x+y\sqrt{x}-y\sqrt{x}-y^2$

$x-y^2$

m) $(\sqrt{x+2}+3)(\sqrt{x+2}-3)$

$x+2-3\sqrt{x+2}+3\sqrt{x+2}-9$

$x-7$

n) $(\sqrt{x+2}+3)(\sqrt{x+2}+3)$

$x+2+3\sqrt{x+2}+3\sqrt{x+2}+9$

$x+11+6\sqrt{x+2}$

o) $(\sqrt{y+1}+\sqrt{y-1})(\sqrt{y+1}-\sqrt{y-1})$

$y+1-\sqrt{y+1}\sqrt{y-1}+\sqrt{y+1}\sqrt{y-1}-(y-1)$

$y+1-y+1=2$

p) $(\sqrt{2x}+\sqrt{y})(\sqrt{2x}-5\sqrt{y})$

$2x-5\sqrt{2xy}+\sqrt{2xy}-5y$

$2x-4\sqrt{2xy}-5y$

q) $(\sqrt{x+1})^2-(\sqrt{x}+1)^2$

$x+1-(\sqrt{x}+1)(\sqrt{x}+1)$

$x+1-(x+\sqrt{x}+\sqrt{x}+1)$

$x+1-x-2\sqrt{x}-1=-2\sqrt{x}$

r) $(\sqrt[3]{x}+1)(\sqrt[3]{x^2}-\sqrt[3]{x}+1)$

$\sqrt[3]{x^3}-\sqrt[3]{x^2}+\sqrt[3]{x}+\sqrt[3]{x^2}-\sqrt[3]{x}+1$

$x+1$

5. Perform the indicated operation and simplify. Assume all variables represent positive real numbers.

a) $\sqrt[2]{x}\cdot\sqrt[3]{x}$ $= x^{\frac{1}{2}}\cdot x^{\frac{1}{3}}$

$x^{\frac{3}{6}}\cdot x^{\frac{2}{6}} = x^{\frac{5}{6}} = \sqrt[6]{x^5}$

b) $\frac{\sqrt{x}}{\sqrt[3]{x}}$ $x^{\frac{1}{2}} \div x^{\frac{1}{3}}$

$x^{\frac{3}{6}-\frac{2}{6}} = x^{\frac{1}{6}} = \sqrt[6]{x}$

c) $\sqrt[2]{x^3}\cdot\sqrt[5]{x^2}$

$x^{\frac{3}{2}+\frac{2}{5}} = x^{\frac{15}{10}+\frac{4}{10}} = x^{\frac{19}{10}}$

$\sqrt[10]{x^{19}}$

d) $\frac{\sqrt[2]{x^3}}{\sqrt[5]{x^2}}$ $x^{\frac{3}{2}-\frac{2}{5}} = x^{\frac{15}{10}-\frac{4}{10}} = x^{\frac{11}{10}}$

$\sqrt[10]{x^{11}}$

e) $\sqrt[4]{a^3}\cdot\sqrt[3]{a^2}$

$a^{\frac{3}{4}+\frac{2}{3}} = a^{\frac{9}{12}+\frac{8}{12}} = a^{\frac{17}{12}}$

$\sqrt[12]{a^{17}}$

f) $\frac{\sqrt{ab^3}}{\sqrt[5]{a^2b^3}}$ $\frac{a^{\frac{1}{2}}b^{\frac{3}{2}}}{a^{\frac{2}{5}}b^{\frac{3}{5}}} = a^{\frac{1}{2}-\frac{2}{5}}b^{\frac{3}{2}-\frac{3}{5}}$

$a^{\frac{5}{10}-\frac{4}{10}}b^{\frac{15}{10}-\frac{6}{10}} = a^{\frac{1}{10}}b^{\frac{9}{10}} = \sqrt[10]{ab^9}$

g) $\sqrt{16x^3y^3}\cdot\sqrt[3]{8xy^2}$

$4x^{\frac{3}{2}}y^{\frac{3}{2}}\cdot 2x^{\frac{1}{3}}y^{\frac{2}{3}}$

$8x^{\frac{3}{2}+\frac{1}{3}}y^{\frac{3}{2}+\frac{2}{3}} = 8x^{\frac{11}{6}}y^{\frac{13}{6}} = 8\sqrt[6]{x^{11}y^{13}}$

$= 8xy^2\sqrt[6]{x^5y}$

h) $\frac{\sqrt[4]{x^2y^3}}{\sqrt{xy}}$ $\frac{x^{\frac{2}{4}}y^{\frac{3}{4}}}{x^{\frac{1}{2}}y^{\frac{1}{2}}} = x^{\frac{2}{4}-\frac{1}{2}}y^{\frac{3}{4}-\frac{1}{2}}$

$= y^{\frac{1}{4}} = \sqrt[4]{y}$

i) $\sqrt{(3-2x)^3}\cdot\sqrt[3]{(3-2x)^2}$ $x \le 3/2$

$(3-2x)^{\frac{3}{2}}(3-2x)^{\frac{2}{3}}$

$(3-2x)^{\frac{3}{2}+\frac{2}{3}} = (3-2x)^{\frac{13}{6}}$

$\sqrt[6]{(3-2x)^{13}} = (3-2x)^2\sqrt[6]{(3-2x)}$

j) $\frac{\sqrt[3]{(1-x)^2}}{\sqrt[4]{1-x}}$ $(1-x)^{\frac{2}{3}-\frac{1}{4}} = (1-x)^{\frac{8-3}{12}}$

$\sqrt[12]{(1-x)^5}$ $x \le 1$

k) $\sqrt[3]{2x-1}\cdot\sqrt[4]{2x-1}$

$(2x-1)^{\frac{1}{3}}(2x-1)^{\frac{1}{4}} = (2x-1)^{\frac{1}{3}+\frac{1}{4}}$

$(2x-1)^{\frac{7}{12}} = \sqrt[12]{(2x-1)^7}$ $x \ge 1/2$

l) $\frac{\sqrt{(2x-1)^4}}{\sqrt[4]{(2x-1)^3}}$ $(2x-1)^{\frac{4}{2}-\frac{3}{4}} = (2x-1)^{\frac{16-6}{8}}$

$(2x-1)^{\frac{10}{8}} = \sqrt[4]{(2x-1)^5}$

$(2x-1)\sqrt[4]{(2x-1)}$ $x \ge 1/2$

m) $\sqrt{x+3}\cdot\sqrt[4]{(x+3)^2}$

$(x+3)^{\frac{1}{2}+\frac{2}{4}} = x+3$

n) $\frac{\sqrt[3]{(2-x)^4}}{\sqrt[4]{(2-x)^3}}$ $(2-x)^{\frac{4}{3}-\frac{3}{4}} = (2-x)^{\frac{16-9}{12}}$

$(2-x)^{\frac{7}{12}} = \sqrt[12]{(2-x)^7}$

$x \le 2$

o) $\sqrt{x}\cdot\sqrt[3]{x^2}\cdot\sqrt[4]{x^3}$

$x^{\frac{1}{2}+\frac{2}{3}+\frac{3}{4}} = x^{\frac{6+8+9}{12}} = x^{\frac{23}{12}}$

$\sqrt[12]{x^{23}} = x\sqrt[12]{x^{11}}$

p) $\frac{\sqrt{x}\cdot\sqrt[3]{x^2}}{\sqrt[4]{x^3}}$ $x^{\frac{1}{2}+\frac{2}{3}-\frac{3}{4}}$

$x^{\frac{6+8-9}{12}} = x^{\frac{5}{12}} = \sqrt[12]{x^5}$

6. Determine what the fraction must be multiplied by to rationalize the denominator.

a) $\frac{1}{\sqrt{x-2}}$ $\sqrt{x-2}$ b) $\frac{1}{\sqrt{x}-\sqrt{2}}$ $\sqrt{x}+\sqrt{2}$

c) $\frac{1}{\sqrt[3]{x-2}}$ $\sqrt[3]{(x-2)^2}$ d) $\frac{\sqrt{x}-2}{\sqrt{x}}$ $\sqrt{x}$

e) $\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}+\sqrt{3}}$ $\sqrt{2}-\sqrt{3}$ f) $\frac{\sqrt{x}-1}{\sqrt{x}+3}$ $\sqrt{x}-3$

g) $\frac{\sqrt{x}}{\sqrt{x}-\sqrt{y}}$ $\sqrt{x}+\sqrt{y}$ h) $\frac{\sqrt{y}}{\sqrt{x}+\sqrt{y}}$ $\sqrt{x}-\sqrt{y}$

7. Simplify, if possible.

a) $\sqrt{2}+\sqrt{5}$ NP b) $\sqrt{2}\cdot\sqrt{5}$ $\sqrt{10}$

c) $\sqrt{6}-\sqrt{3}$ NP d) $\frac{\sqrt{6}}{\sqrt{3}}$ $\sqrt{2}$

e) $\sqrt{3}-2\sqrt{3}$ $-\sqrt{3}$ f) $\sqrt{3}\cdot 2\sqrt{3} = 2\sqrt{9} = 2(3)$ 6

g) $\frac{\sqrt{3}}{4\sqrt{3}}$ $\frac{1}{4}$ h) $\frac{\sqrt{3}}{4\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right) = \frac{\sqrt{6}}{8}$ ____

i) $\frac{1}{\sqrt[3]{3}}\left(\frac{\sqrt[3]{3^2}}{\sqrt[3]{3^2}}\right)$ $\frac{\sqrt[3]{3^2}}{3} = \frac{\sqrt[3]{9}}{3}$ j) $\frac{1}{\sqrt[4]{2}}\left(\frac{\sqrt[4]{2^3}}{\sqrt[4]{2^3}}\right) = \frac{\sqrt[4]{8}}{2}$ ____

8. Mentally find each product. Assume all variables are positive.

a) $(\sqrt{5}+2)(\sqrt{5}-2)$

$5-4=1$

b) $(\sqrt{3}-\sqrt{2})(\sqrt{3}+\sqrt{2})$

$3-2=1$

c) $(4+\sqrt{3})(4-\sqrt{3})$

$16-3=13$

d) $(\sqrt{x}-y)(\sqrt{x}+y)$

$x-y^2$

e) $(\sqrt{x}+\sqrt{y})(\sqrt{x}-\sqrt{y})$

$x-y$

f) $(\sqrt{x+1}-1)(\sqrt{x+1}+1)$

$x+\cancel{1}-\cancel{1}=x$

g) $(\sqrt{x+2}-\sqrt{y+2})(\sqrt{x+2}+\sqrt{y+2})$

$x+\cancel{2}-(y+\cancel{2})=x-y$

h) $(x-\sqrt{x-1})(x+\sqrt{x-1})$

$x^2-(x-1)=x^2-x+1$

i) $(\sqrt{1-x}-\sqrt{x-1})(\sqrt{1-x}+\sqrt{x-1})$

$1-x-(x-1)$

$=2-2x$

j) $(\sqrt{2x+3}+\sqrt{2x-3})(\sqrt{2x+3}-\sqrt{2x-3})$

$2x+3-(2x-3)$

$=6$

9. Rationalize the denominator. Write the quotient in lowest terms. Assume all variables are positive.

a) $\frac{1}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{\sqrt{2}}{2}$

b) $\frac{1}{\sqrt[3]{2}}\left(\frac{\sqrt[3]{2^2}}{\sqrt[3]{2^2}}\right)=\frac{\sqrt[3]{4}}{2}$

c) $\frac{3+\sqrt{2}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)=\frac{3\sqrt{2}+2}{2}$

d) $\frac{5-\sqrt{2}}{\sqrt{3}}\left(\frac{\sqrt{3}}{\sqrt{3}}\right)=\frac{5\sqrt{3}-\sqrt{6}}{3}$

e) $\frac{1}{3+\sqrt{2}}\left(\frac{3-\sqrt{2}}{3-\sqrt{2}}\right)=\frac{3-\sqrt{2}}{7}$

$9-2$

f) $\frac{1}{3-\sqrt{2}}\left(\frac{3+\sqrt{2}}{3+\sqrt{2}}\right)=\frac{3+\sqrt{2}}{7}$

$9-2$

g) $\frac{\sqrt{12}}{\sqrt{3}+1}\left(\frac{\sqrt{3}-1}{\sqrt{3}-1}\right)=\frac{2(3)-2\sqrt{3}}{2}$

$2\sqrt{3}$

$3-1$

$=3-\sqrt{3}$

h) $\frac{\sqrt{18}}{\sqrt{2}-1}\left(\frac{\sqrt{2}+1}{\sqrt{2}+1}\right)=3(2)+3\sqrt{2}$

$3\sqrt{2}$

$2-1$

$=6+3\sqrt{2}$

i) $\frac{3+\sqrt{2}}{1+\sqrt{2}}\left(\frac{1-\sqrt{2}}{1-\sqrt{2}}\right)=\frac{3-3\sqrt{2}+\sqrt{2}-2}{-1}$

$1-2$

$=\frac{1-2\sqrt{2}}{-1}=2\sqrt{2}-1$

j) $\frac{\sqrt{5}}{\sqrt{2}-\sqrt{3}}\left(\frac{\sqrt{2}+\sqrt{3}}{\sqrt{2}+\sqrt{3}}\right)=\frac{\sqrt{10}+\sqrt{15}}{-1}=-\sqrt{10}-\sqrt{15}$

$2-3$

k) $\frac{1}{\sqrt{x}-\sqrt{y}}\left(\frac{\sqrt{x}+\sqrt{y}}{\sqrt{x}+\sqrt{y}}\right)=\frac{\sqrt{x}+\sqrt{y}}{x-y}$

l) $\frac{\sqrt{a}+b}{\sqrt{a}-b}\left(\frac{\sqrt{a}+b}{\sqrt{a}+b}\right)=\frac{a+b\sqrt{a}+b\sqrt{a}+b^2}{a-b^2}$

$a-b^2$

$=\frac{a+2b\sqrt{a}+b^2}{a-b^2}$

10. The volume of a cone is $V = \frac{1}{3}\pi r^2 h$. If the volume of a cone is 18π cm³ and the height is 6 cm, what is the radius?

$18\pi = \frac{1}{3}\pi r^2(6)$

$18 = 2r^2$

$9 = r^2$

$r = 3\text{cm}$

11. The surface area of a sphere is $S = 4\pi r^2$. If the surface area of the sphere is 144 mm², what is the radius?

$144 = 4\pi r^2$

$36 = \pi r^2$

$\frac{36}{\pi} = r^2$

$r = \sqrt{\frac{36}{\pi}}$ → $r = \frac{6}{\sqrt{\pi}}\left(\frac{\sqrt{\pi}}{\sqrt{\pi}}\right)$

$r = \frac{6\sqrt{\pi}}{\pi}$ mm.

12. The volume of a sphere is $V = \frac{4}{3}\pi r^3$. If the volume of a sphere is 36 cm³, what is the radius?

$36 = \frac{4}{3}\pi r^3$

$\frac{3}{4}(36) = \pi r^3$

$27 = \pi r^3$

$\frac{27}{\pi} = r^3$

$r = \sqrt[3]{\frac{27}{\pi}}$ → $r = \frac{3}{\sqrt[3]{\pi}}\left(\frac{\sqrt[3]{\pi^2}}{\sqrt[3]{\pi^2}}\right)$

$r = \frac{3\sqrt[3]{\pi^2}}{\pi^2}$ cm

13. A pendulum of length l metres takes $t = 2\pi\sqrt{\frac{l}{g}}$ seconds to complete a swing cycle. If the swing cycle is two seconds and g, the acceleration due to gravity is 9.8 m/s², how long is the pendulum?

$1 = 2\pi\sqrt{\frac{l}{9.8}}$

$\frac{1}{\pi} = \sqrt{\frac{l}{9.8}}$

$\frac{1}{\pi^2} = \frac{l}{9.8}$

$\frac{9.8}{\pi^2}\text{ m} = l$

14. The volume of a pyramid is $V = \frac{1}{3}B \cdot h$, where B is the area of the base. If the volume of a square based pyramid is 300 m³ and the height is 9 m, what is the length of one side of the base?

$300 = \frac{1}{3}B(9)^3$

$100 = B$

$\sqrt{100} = \sqrt{B}$

∴ one side of the base = 10 m.

15. The distance formula of a falling object is given by $d = \frac{1}{2}gt^2$. What time does it take for an object to fall 40 m? ($g = 9.8$ m/s²)

$40 = \frac{1}{2}(9.8)t^2$

$40 = 4.9t^2$

$\frac{40}{4.9} = t^2$ ← $\frac{400}{49}$

$\sqrt{\frac{400}{49}} = t$ → $t = \frac{20}{7}$ s.

16. The distance to the horizon on the ocean is approximated by the formula $h = \frac{2}{3}d^2$, where d is miles to the horizon and h is number of feet above sea level. Standing on the observation deck 25 feet above sea level, how far can you see?

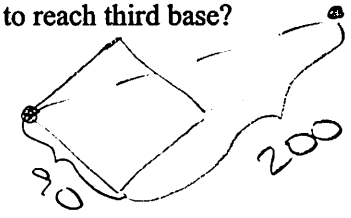
$25 = \frac{2}{3}d^2$

$\frac{75}{2} = d^2$

$\sqrt{\frac{75}{2}} = d$ → $d = \frac{5\sqrt{3}}{\sqrt{2}}\left(\frac{\sqrt{2}}{\sqrt{2}}\right)$

$d = \frac{5\sqrt{6}}{2}$ miles

17. A baseball diamond is a square with sides 90 ft. If a ball is caught on the right field line 200 ft from home plate, what distance must the fielder throw the ball to reach third base?



$d = \sqrt{90^2 + 200^2}$

$d = \sqrt{48100}$

$d = 10\sqrt{481}$ ft.

1.6 Exercise Set

1. Square each of the following expressions.

a) $\sqrt{x+2}$ $x+2$ b) $\sqrt{x}+2$ $x+4\sqrt{x}+4$

c) $\sqrt{3x-5}$ $3x-5$ d) $\sqrt{3x}-5$ $3x-10\sqrt{3x}+25$

e) $\sqrt{1-4x}$ $1-4x$ f) $1-4\sqrt{x}$ $1-8\sqrt{x}+16x$

g) $x-3$ x^2-6x+9 h) $\sqrt{x}-\sqrt{3}$ $x-2\sqrt{3x}+3$

2. Determine the restrictions on the following radical equations.

a) $\sqrt{x+5}=4$ $x \geq -5$ b) $\sqrt{9-x}=5$ $x \leq 9$

c) $\sqrt{2x+3}=6$ $x \geq -3/2$ d) $\sqrt{10x-8}=3\sqrt{x}$ $x \geq 4/5$
$x \geq \frac{8}{10}$ $x \geq 0$.

e) $\sqrt{5x-5}=\sqrt{4x-1}$ $x \geq 1$ f) $\sqrt{3x+3}=\sqrt{5x-1}$ $x \geq 1/5$.
$x \geq 1$ $x \geq \frac{1}{4}$ $x \geq -1$, $x \geq \frac{1}{5}$

g) $\sqrt{x}+6=\sqrt{x+72}$ $x \geq 0$ h) $\sqrt{10-3x}=\sqrt{2x+20}$ $-10 \leq x \leq 10/3$
$x \geq 0$ $x \geq -72$ $x \leq 10/3$ $x \geq -10$.

i) $\sqrt{2x-1}=\sqrt{1-2x}$ $x = 1/2$ j) $\sqrt{1-2x}=\sqrt{x+10}$ $-10 \leq x \leq 1/2$
$x \geq 1/2$ $x \leq 1/2$ $x \leq 1/2$ $x \geq -10$

3. Solve the following radical equations.

a) $(\sqrt{2t-3})^2 = 5^2$ $t \geq 3/2$

$2t-3=25$

$2t=28$

$t=14$

b) $\sqrt{3t+4} = -2$ NP $\varnothing$ $t \geq -4/3$

c) $\sqrt{1-3x} = -2$ NP $\varnothing$

d) $2\sqrt{x-1} = x$ $x \geq 1$

$4(x-1) = x^2$

$4x-4 = x^2$

$x^2-4x+4=0$

$(x-2)^2=0$

$x=2$

$x \geq -3/2$ $x \geq -2$

e) $\sqrt{2x+3} - \sqrt{x+2} = 2$

$(\sqrt{2x+3})^2 = (\sqrt{x+2}+2)^2$

$2x+3 = x+2+4\sqrt{x+2}+4$

$(x-3)^2 = (4\sqrt{x+2})^2$

$x^2-6x+9 = 16(x+2)$

$x^2-22x-23=0$

$(x-23)(x+1)=0$

$x=23, \cancel{-1}$

f) $-\sqrt{x}+3 = \sqrt{3x+5}$ NP $\varnothing$

g) $(\sqrt{2x+1})^2 = (x-7)^2$

$2x+1 = x^2-14x+49$

$x^2-16x+48=0$

$(x-12)(x-4)=0$

$x=12, \cancel{4}$

h) $(\sqrt{3x+10})^2 + 5 = (2x-5)^2$

$3x+10 = 4x^2-20x+25$

$4x^2-23x+15=0$

$(4x-3)(x-5)=0$

$x=\cancel{3/4}, 5$

i) $(x+3)^2 = (\sqrt{x+1})(\sqrt{x+6})^2$

$x^2+6x+9 = (x+1)(x+6)$

$x^2+6x+9 = x^2+7x+6$

$3 = x$

j) $(\sqrt{y-8})^2 + \sqrt{y} = (2-\sqrt{y})^2$

$y-8 = 4-4\sqrt{y}+y$

$-12 = -4\sqrt{y}$

$3 = \sqrt{y}$

$\cancel{9} = y$ $\varnothing$ no solution

k) $\sqrt{1-x} + \sqrt{x+9} = 4$ $(\sqrt{1-x})^2 = (4-\sqrt{x+9})^2$

$1-x = 16-8\sqrt{x+9}+x+9$

$(-24-2x)^2 = (-8\sqrt{x+9})^2$

$576+96x+4x^2 = 64(x+9)$

$4x^2+32x=0$

$4x(x+8)=0$ $x=0,-8$

l) $(\sqrt{2x+11})^2 + \sqrt{x+6} = (2-\sqrt{x+6})^2$

$2x+11 = 4-4\sqrt{x+6}+x+6$

$(x+1)^2 = (-4\sqrt{x+6})^2$

$x^2+2x+1 = 16(x+6)$

$x^2-14x-95=0$

$(x-19)(x+5)=0$

$x=\cancel{19}, -5$

m) $\sqrt{z+6} = \frac{2}{\sqrt{z+1}} + \sqrt{z+1}$

$(\sqrt{z+1}\sqrt{z+6})^2 = (2+z+1)^2$

$(z+1)(z+6) = z^2+6z+9$

$z^2+7z+6 = z^2+6z+9$

$z=3$

n) $\sqrt{x+15} + \sqrt{x+7} = \frac{4}{\sqrt{x+7}}$

$\sqrt{x+15}\sqrt{x+7} + x+7 = 4$

$(\sqrt{x+15}\sqrt{x+7})^2 = (-x-3)^2$

$(x+15)(x+7) = x^2+6x+9$

$x^2+22x+105 = x^2+6x+9$

$16x = -96$ $x=-6$

4. The formula $V = \sqrt{2gh}$ relates velocity, V, in metres per second of an object after h metres accelerated by gravity, g, in metres per second squared. If g is approximately 9.8 m/s², how far has an object fallen if its velocity is 30 m/s?

$30 = \sqrt{2(9.8)h}$

$30 = \sqrt{19.6h}$

$30^2 = 19.6h$

$\frac{30^2}{19.6} = h$

$h = \frac{2250}{49}\ m. = 45.9m$

5. The maximum distance, d, in kilometres that a person can see from a height, h, in kilometres above the ground is $d = 111.7\sqrt{h}$. Find the height in metres that would allow a person to see 75 kilometres.

$75 = 111.7\sqrt{h}$

$\frac{75}{111.7} = \sqrt{h}$

$\left(\frac{75}{111.7}\right)^2 = h$

$h = 0.451km \rightarrow 451\ m.$

6. Heron's formula for finding the area of a triangle with sides a, b, c is $A = \sqrt{s(s-a)(s-b)(s-c)}$, where $s = (a+b+c)/2$. Find the area of a triangle with sides 6, 8, and 12 cm.

$s = (6+8+12)/2 = 13$

$A = \sqrt{13(13-6)(13-8)(13-12)}$

$A = \sqrt{13 \cdot 7 \cdot 5 \cdot 1} = \sqrt{455}$

$= 21.33cm$

7. A pendulum of length l metres takes $t = 2\pi\sqrt{\ell/g}$ seconds to go through a complete cycle. If a pendulum takes three seconds to make a complete cycle, how long is the pendulum? ($g = 9.8$ m/s²)

$3 = 2\pi\sqrt{\frac{l}{9.8}}$

$\left(\frac{3}{2\pi}\right)^2 = \left(\sqrt{\frac{l}{9.8}}\right)^2$

$\frac{9}{4\pi^2} = \frac{l}{9.8}$

$l = \frac{9.8 \cdot 9}{4\pi^2} = 2.23m.$

8. The perimeter of an elliptical garden is given by $P = 2\pi\sqrt{(a^2+b^2)/2}$, where a is the width of and b is the height of the ellipse. Determine a if $b = 6$ cm and $P = 20\pi$ cm.

$20\pi = 2\pi\sqrt{(a^2+6^2)/2}$

$(10)^2 = \left(\sqrt{\frac{a^2+36}{2}}\right)^2$

$100 = \frac{a^2+36}{2}$

$200 = a^2+36$

$164 = a^2$

$\sqrt{164} = a$

$a = 12.81cm$

9. The kinetic energy of an object is given by the formula $E = \frac{1}{2}mv^2$, where m is the mass in kilograms, v is the speed in metres per second, and E is the kinetic energy in joules. If the kinetic energy is 12 960 joules and the mass is 80 kg, what speed is the object travelling at?

$12960 = \frac{1}{2} \cdot 80 \cdot v^2$

$324 = v^2$

$\sqrt{324} = v$

$v = 18\ m/s.$

1.7 Chapter Review

Section 1.1

1. Evaluate.

a) $|-3|-|-2|$ 1

b) $-|-3-(-4)|$ -1

c) $-|2|-|-2|$ -4

d) $|-4|+(-4)$ 0

Section 1.2

2. Solve for x.

a) $x^2 = 16$ $x = \pm 4$

b) $x^3 = 27$ $x = 3$

c) $x^2 = -16$ $\varnothing$

d) $x^3 = -27$ $x = -3$

e) $x^6 = 3$ $x = \pm\sqrt[6]{3}$

f) $x^5 = 3$ $x = \sqrt[5]{3}$

g) $x^6 = -3$ $\varnothing$

h) $x^5 = -3$ $x = \sqrt[5]{-3}$

3. Simplify. Let the variables be any real number.

a) $\sqrt{9x^7}$ $3x^3\sqrt{x}$, $x \geq 0$

b) $\sqrt{9x^6}$ $3x^3$

c) $\sqrt[3]{-x^9}$ $-x^3$

d) $\sqrt[3]{(3-x)^6}$ $(3-x)^2$

e) $\sqrt[4]{x^7y^8}$ $x^2y^2\sqrt[4]{x^3}$, $x \geq 0$

f) $\sqrt[4]{(x+1)^5}$ $(x+1)\sqrt[4]{(x+1)}$, $x \geq -1$

Section 1.3

4. Simplify. All variables represent non-negative real numbers.

a) $\sqrt{75} = \sqrt{25 \times 3}$ $5\sqrt{3}$

b) $\sqrt[3]{40} = \sqrt[3]{8 \times 5}$ $2\sqrt[3]{5}$

c) $-\frac{3}{4}\sqrt{80} = \frac{-3\sqrt{16 \times 5}}{4}$ $-3\sqrt{5}$

d) $\sqrt[3]{-250} = \sqrt[3]{-125 \times 2}$ $-5\sqrt[3]{2}$

e) $\sqrt[4]{81x^6y^7}$ $3x^2y\sqrt[4]{x^2y^3}$

f) $\sqrt[3]{-16x^3y^4} = \sqrt[3]{-8 \times 2x^3y^4}$ $-2xy\sqrt[3]{2y}$

5. Write as an entire radical. All variables represent non-negative real numbers.

a) $3\sqrt{5} = \sqrt{3^2 \cdot 5}$ $\sqrt{45}$ b) $3\sqrt[3]{5} = \sqrt[3]{3^3 \cdot 5}$ $\sqrt[3]{135}$

c) $3\sqrt[4]{5} = \sqrt[4]{3^4 \cdot 5}$ $\sqrt[4]{405}$ d) $3\sqrt[5]{5} = \sqrt[5]{3^5 \cdot 5}$ $\sqrt[5]{1215}$

e) $-2xy^2\sqrt{x^2y^3}$ $-\sqrt{4x^4y^7}$ f) $-2xy^2\sqrt[3]{x^2y^4}$ $\sqrt[3]{-8x^5y^{10}}$

Section 1.4

6. Simplify. All variables represent positive numbers.

a) $\sqrt{28} + \sqrt{175}$ $2\sqrt{7} + 5\sqrt{7}$ $7\sqrt{7}$ b) $\sqrt{32} - \sqrt{98}$ $4\sqrt{2} - 7\sqrt{2}$ $-3\sqrt{2}$

c) $5\sqrt{8x} - 3\sqrt{18x}$ $5(2)\sqrt{2x} - 3(3)\sqrt{2x}$ $\sqrt{2x}$ d) $x\sqrt{18} + 4x\sqrt{8} - 5x\sqrt{3}$ $x(3)\sqrt{2} + 4x(2)\sqrt{2} - 5x\sqrt{3}$ $11x\sqrt{2} - 5x\sqrt{3}$

e) $3x\sqrt{20x} - \sqrt{24x} + \sqrt{45x^3}$ $3x(2)\sqrt{5x} - 2\sqrt{6x} + 3x\sqrt{5x}$ $9x\sqrt{5x} - 2\sqrt{6x}$ f) $\sqrt{\frac{28}{2x}} + \sqrt{\frac{112}{x^2}}$ $\frac{2\sqrt{7}}{2x} + \frac{4\sqrt{7}}{x}$ $\frac{5\sqrt{7}}{x}$

g) $\sqrt[3]{40} + 4\sqrt[3]{625}$ $2\sqrt[3]{5} + 4(5)\sqrt[3]{5}$ $22\sqrt[3]{5}$ h) $3\sqrt[3]{16} - \sqrt[3]{54}$ $3 \cdot 2\sqrt[3]{2} - 3\sqrt[3]{2}$ $3\sqrt[3]{2}$

i) $\sqrt[3]{24x} + \sqrt[3]{3x} - \sqrt[3]{375x}$ $2\sqrt[3]{3x} + \sqrt[3]{3x} - 5\sqrt[3]{3x}$ $-2\sqrt[3]{3x}$ j) $\sqrt[3]{x^7} - 2x\sqrt[3]{x^4} + 3x^2\sqrt[3]{x}$ $x^2\sqrt[3]{x} - 2x(x)\sqrt[3]{x} + 3x^2\sqrt[3]{x}$ $2x^2\sqrt[3]{x}$

Section 1.5

7. Find the indicated operation and simplify. All variables represent positive numbers.

a) $3\sqrt{2}(5\sqrt{7} - 4\sqrt{3})$ $15\sqrt{14} - 12\sqrt{6}$ $15\sqrt{14} - 12\sqrt{6}$ b) $3\sqrt[3]{2}(2\sqrt[3]{4} - 4\sqrt[3]{2})$ $6\sqrt[3]{8} - 12\sqrt[3]{4}$ $6 \cdot 2 - 12\sqrt[3]{4}$ $12 - 12\sqrt[3]{4}$

c) $(2-\sqrt{3})(2+\sqrt{3})$ $4 - 3$ 1 d) $(2-\sqrt{3})^2$ $4 - 4\sqrt{3} + 3$ $7 - 4\sqrt{3}$

e) $(\sqrt[3]{9} - 1)(\sqrt[3]{3} + 1)$ $\sqrt[3]{27} + \sqrt[3]{9} - \sqrt[3]{3} - 1$ $2 + \sqrt[3]{9} - \sqrt[3]{3}$ f) $(\sqrt{x} + 2\sqrt{5})(\sqrt{x} - 2\sqrt{5})$ $x - 4(5)$ $x - 20$

g) $(\sqrt{x} + 2\sqrt{5})^2$ $x + 4\sqrt{5x} + 4(5)$ $x + 20 + 4\sqrt{5x}$ h) $(\sqrt[3]{x^2} - 1)(\sqrt[3]{x^4} + \sqrt[3]{x^2} + 1)$ $\sqrt[3]{x^6} + \sqrt[3]{x^4} + \sqrt[3]{x^2} - \sqrt[3]{x^4} - \sqrt[3]{x^2} - 1$ $x^2 - 1$

i) $\sqrt[2]{x} \cdot \sqrt[3]{x} \cdot \sqrt[4]{x}$ $x^{\frac{1}{2}+\frac{1}{3}+\frac{1}{4}} = x^{\frac{13}{12}}$ $\sqrt[12]{x^{13}}$ $\hookrightarrow x\sqrt[12]{x}$ j) $\frac{\sqrt[2]{x^5}}{\sqrt[3]{x^2}}$ $x^{\frac{5}{2}-\frac{2}{3}} = x^{\frac{11}{6}}$ $\sqrt[6]{x^{11}}$ $\hookrightarrow x\sqrt[6]{x^5}$

8. Rationalize the denominator.

a) $\frac{1}{\sqrt{5}}\left(\frac{\sqrt{5}}{\sqrt{5}}\right)$ $\frac{\sqrt{5}}{5}$

b) $\frac{1}{\sqrt[3]{5}}\frac{\sqrt[3]{5^2}}{\sqrt[3]{5^2}}$ $\frac{\sqrt[3]{25}}{5}$

c) $\frac{1}{2-\sqrt{3}}\left(\frac{2+\sqrt{3}}{2+\sqrt{3}}\right)$ $2+\sqrt{3}$

$4-3$

d) $\frac{\sqrt{x}+y}{\sqrt{x}-y}\left(\frac{\sqrt{x}+y}{\sqrt{x}+y}\right)$ $\frac{x+2y\sqrt{x}+y^2}{x-y^2}$

e) $\frac{\sqrt{x}-\sqrt{2y}}{\sqrt{x}+\sqrt{2y}}\left(\frac{\sqrt{x}-\sqrt{2y}}{\sqrt{x}-\sqrt{2y}}\right)$ $\frac{x-2\sqrt{2xy}+2y}{x-2y}$

f) $\frac{\sqrt{2}x-y}{\sqrt{2}x+y}\left(\frac{\sqrt{2}x-y}{\sqrt{2}x-y}\right)$ $\frac{2x^2-2xy\sqrt{2}+y^2}{2x^2-y^2}$

Section 1.6

9. Solve the following radical equations.

a) $\left(\sqrt{x+1}\right)^2=(x+1)^2$ $x=0,-1$

$x+1=x^2+2x+1$

$0=x^2+x$

$0=x(x+1)$

$x=0,-1$

b) $\left(\sqrt{x-5}\right)^2=\left(\sqrt{x}-1\right)^2$ $x=9$

$x-5=x-2\sqrt{x}+1$

$-6=-2\sqrt{x}$

$3=\sqrt{x}$

$9=x$

c) $\left(\sqrt{x+1}\right)^2=\left(\sqrt{x}+1\right)^2$ $x=0$

$x+1=x+2\sqrt{x}+1$

$0=2\sqrt{x}$

$x=0$

d) $\left(\sqrt{x-4}\right)^2=\left(2+\sqrt{x}\right)^2$ no solutions.

$x-4=4+4\sqrt{x}+x$

$-8=+4\sqrt{x}$

$-2=\sqrt{x}$

$4=x$

e) $\left(\sqrt{2x-1}\right)^2+\sqrt{x+3}=\left(3-\sqrt{x+3}\right)^2$ $x=$

$2x-1=9-6\sqrt{x+3}+x+3$

$(x-13)^2=-(6\sqrt{x+3})^2$

$x^2-26x+169=36(x+3)$

$x^2-62x+61=0$ $x=61,1$ $(x-61)(x-1)=0$

f) $\left(\sqrt{2x-1}\right)^2=\left(\sqrt{x-4}+2\right)^2$

$2x-1=x-4+4\sqrt{x-4}+4$

$(x-1)^2=\left(4\sqrt{x-4}\right)^2$

$x^2-2x+1=16(x-4)$

$x^2-18x+65=0$

$(x-13)(x-5)=0$ $x=13,5$

g) $\sqrt{x+4}+\sqrt{3x+9}=\sqrt{x+25}$

$x+4+2\sqrt{x+4}\cdot\sqrt{3x+9}+3x+9=x+25$

$4x+13+2\sqrt{x+4}\sqrt{3x+9}=x+25$

$2\sqrt{x+4}\sqrt{3x+9}=-3x+12$

$4(x+4)(3x+9)=9x^2-72x+144$

$4(3x^2+21x+36)=9x^2-72x+144$

h) $\sqrt{x-3}+\sqrt{2x+1}=2\sqrt{x}$

$x-3+2\sqrt{x-3}\sqrt{2x+1}+2x+1=4x$

$2\sqrt{x-3}\sqrt{2x+1}=x+2$

$4(x-3)(2x+1)=x^2+4x+4$

$4(2x^2-5x-3)=x^2+4x+4$

$7x^2-24x-16=0$

$(7x+4)(x-4)=0$ $x=-\frac{4}{7}$

$3x^2+15x=0$

$3x(x+5)=0$

$x=0,-5$ $x=0$